The Non-Chaotic Quasi Periodic Behavior of Food Web of Two Logistic Prey And Modified Leslie- Gower Type Predator with Switching

Brahampal Singh
Associate Professor,
Deptt.ofMathematics,
J.V.Jain(PG) College,
Saharanpur, U.P.

SunitaGakkhar
Professor,
Deptt.of Mathematics,
Indian Institute of Technology,
Roorkee

Abstract

In this research paper we study the mathematical model numerically taken from published research paper [1-2]. Modified Leslie-Gower predator has switching with prey species if one prey species goes to massive loss due to predating. Through numerical simulations the loss of stability is observed in range of biological feasible values for the key parameters. The persistence in the form of non-chaotic quasiperiodic solution is also observed. This paper is the extended part of published research paper [2].

Keywords: Range of Biological Feasible Parameters, Quasiperiodic Attractor, Switching Effect of Predator to Prey Species.

Introduction

We study various research papers [1-23] that the nature is nonlinear which shown multi rich dynamics. In this research paper for the given biological feasible range of parameters under Kolmogorov conditions is observed non-chaotic quasiperiodic solution and switching solution of the mathematical model.

Review of Literature

A lot of research work has been carried out on ecological systems comprising of food chains and food web of variable lengths [03-22]. The nonlinear equations of mathematical model have local, global stability and hopf bifurcation [1-2].

Aim to Study

We have to study numerically quasiperiodic non-chaotic solutions and switching behavior of the predator to prey species under the range of biological parameters of the nonlinear model [1-2].

The Mathematical Model

Consider two prey one predator food web system. Two prey species are assumed to grow logistically. The predator dynamics is assumed to be of modified Leslie- Gower type. The Mathematical model is given by the following non-linear system of equations [1-2]:

$$\frac{dX_1}{dt} = r_1 X_1 (1 - \frac{X_1}{K}) - \frac{A_1 X_1 X_3}{1 + B_1 X_1 + B_2 X_2}$$

$$\frac{dX_2}{dt} = r_2 X_2 (1 - \frac{X_2}{K}) - \frac{A_2 X_3 X_2}{1 + B_1 X_1 + B_2 X_2}$$

$$\frac{dX_3}{dt} = r_3 {X_3}^2 (1 - \frac{1}{S_3 + S_1 X_1 + S_2 X_2})$$

$X_i \geq 0, i = 1,2$ represent the population density of two preys and $X_3 \geq 0$ is the population density of the predator. The constants K_i, r_i, A_i, B_i and S_i, are model parameters assuming only positive values. In the model, the third equation is written according to the Leslie-Gower scheme in which the conventional carrying capacity term is being replaced by the renewable resources for the predator as $S_1 X_1 + S_2 X_2$. The following dimensionless variables/ and parameters are introduced:

$t = r_1 T,\ y_i = X_i / K_i\ ,\ y_3 = X_3 / K_1, w_1 = A_1 / r_1,\ w_2 = B_1 K_1\ , w_3 = B_2 K_2, w_4 = r_2 / r_1,$

$w_5 = A_2 K_1 / r_1, w_6 = r_3 K_1 / r_1, w_7 = 1/S_3\ , w_8 = \alpha_1 w_2\ , w_9 = \alpha_2 w_3, \alpha_1 = S_1 / S_3 B_1, \alpha_2 = S_2 / S_3 B_2$

The transformed non-dimensional form of the biological food web [1] is given

$$\frac{dy_1}{dt} = y_1(1 - y_1 - \frac{w_1 y_3}{1 + w_2 y_1 + w_3 y_2}) = y_1 f_1(y_1, y_2, y_3)$$

$$\frac{dy_2}{dt} = y_2[(1 - y_2)w_4 - \frac{w_5 y_3}{1 + w_3 y_2 + w_2 y_1}] = y_2 f_2(y_1, y_2, y_3) \quad (2)$$

$$\frac{dy_3}{dt} = w_6 y_3^2 (1 - \frac{w_7}{1 + w_8 y_1 + w_9 y_2}) = w_6 y_3^2 (1 - \frac{w_7}{1 + \alpha_1 w_2 y_1 + \alpha_2 w_3 y_2}) = y_3 f_3(y_1, y_2, y_3)$$

$$w_i > 0, i = 1,2,3,4,5,6,7;\ \ y_i \geq 0,\ i = 1,2,3;\ \alpha_1 \neq \alpha_2 .$$

Theorem: Consider the domain $D = \{(y_1, y_2, y_3) : 0 < y_1 < \bar{y}_1 < 1, 0 < y_2 < \bar{y}_2 < 1, 0 < y_3\}$ under the Kolmogorov biological conditions. Then the mathematical model (2) is bounded in D.

Proof:

$$\frac{dy_1}{dt} = y_1(1 - y_1 - \frac{w_1 y_3}{1 + w_2 y_1 + w_3 y_2}) \Rightarrow y_1(t) \leq 1$$

$$\frac{dy_2}{dt} = y_2[(1 - y_2)w_4 - \frac{w_5 y_3}{1 + w_3 y_2 + w_2 y_1})] \Rightarrow y_2(t) \leq 1$$

$$\frac{d}{dt}[y_1(t) + w_4 y_2(t) + w_6 y_3(t)] \leq Max\{y_1(1 - y_1)\} + Max\{y_2(1 - y_2)\} w_4^2 + Max\{w_6^2 y_3^2 (1 - \frac{w_7}{1 + w_8 y_1 + w_9 y_2})\}$$

$$\frac{d}{dt}[y_1(t) + w_4 y_2(t) + w_6 y_3(t)] < \frac{1}{4} + \frac{w_4^2}{4} + M = M_1(say) \Rightarrow y_1(t) + w_4 y_2(t) + w_6 y_3(t) < M_2(say)$$

Mathematical Analysis

The system can be splitted into two disconnected Kolmogorov food sub webs [1]

Lemma 1

Consider the domain $D_1 = \{(y_1, y_3) : 0 < y_1 < \bar{y}_1 < 1, 0 < y_3\}$ and $D_2 = \{(y_2, y_3) : 0 < y_2 < \bar{y}_2 < 1, 0 < y_3\}$, he sub webs of (2) in [1] is Kolmogorov in the domain D1 and domain D2 under the following conditions:

$$w_7 / (1 + \alpha_1 w_2 \bar{y}_1)^2 < 1 < w_7 / (1 + \alpha_1 w_2 \bar{y}_1) < w_7 \quad (3)$$

$$w_7 / (1 + \alpha_2 w_3 \bar{y}_2)^2 < 1 < w_7 / (1 + \alpha_2 w_3 \bar{y}_2) < w_7 \text{ respectively} \quad (4)$$

The proof of the theorem given for existence of positive equilibrium point and stability is established in [1]

Theorem

The system (2) has positive equilibrium point $(\hat{y}_1, \hat{y}_2, \hat{y}_3)$ under (3) and (4) provided one of the following is satisfied:

$$w_3(w_1 w_4 - w_5) < \frac{w_1 w_4 \varepsilon}{\alpha_2};\ \varepsilon = w_7 - 1 \quad (5)$$

$$w_2(w_5 - w_1 w_4) < \frac{\varepsilon w_5}{\alpha_1} \quad (6)$$

Theorem: The positive equilibrium point $(\hat{y}_1, \hat{y}_2, \hat{y}_3)$ is locally asymptotically stable provided the following are satisfied simultaneously:

$$2(\ w_1 w_4 \varepsilon + w_5 w_3 \alpha_2\ ) + \frac{\Delta}{w_2} > \Delta + 2\alpha_2 w_3 w_1 w_4 \quad (7)$$

$$w_3(w_5 \varepsilon + \alpha_1 w_1 w_4 w_2\ ) + \Delta > \frac{w_3 w_5}{w_1 w_4}\Delta + \alpha_1 w_2 w_5 w_3 \quad (8)$$

$$\varepsilon = w_7 - 1 > 0; \quad \Delta = \alpha_1 w_1 w_2 w_4 + \alpha_2 w_3 w_5$$

The following theorem [1] gives the conditions for the global stability of positive nonzero equilibrium point.

Theorem

The positive equilibrium point $(\hat{y}_1, \hat{y}_2, \hat{y}_3)$ is globally asymptotically stable provided the following are satisfied:

$$A = (1 + w_2\hat{y}_1 + w_3\hat{y}_2 - w_2) > 0; \ B = (1 + w_2\hat{y}_1 + w_3\hat{y}_2 -$$

$$w_3^2 m^2 + w_2^2 w_4^2 < 4 m w_4 AB\,;$$

$$m = \frac{\alpha_1 w_2 w_5}{\alpha_2 w_1 w_3} \quad (9)$$

Numerical Simulation

The numerical values for various parameters are selected according to the mathematical restrictions obtained from the Kolmogorov analysis [1-2]. These ensure that the parameters take biologically relevant values only [1-2]. As the solution of the system is bounded, the long time behavior of the solution is obtained as limit cycle attractor, quasi-periodic or non-chaotic attractor.

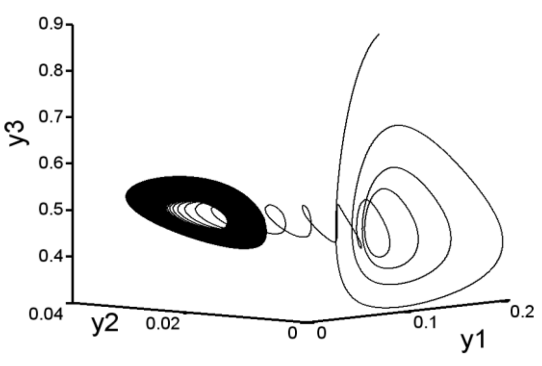
For the following data the two subsystems are Kolmogorov and both of them admits stable limit points in respectiveplanes:

$$w_1 = 3.3, w_2 = 1.2, w_3 = 1.3, w_4 = 1.1, w_5 = 2.5,$$
$$w_6 = 1.0, w_7 = 1.41, a_1 = 3.5, a_2 = 2.5 \quad (10)$$

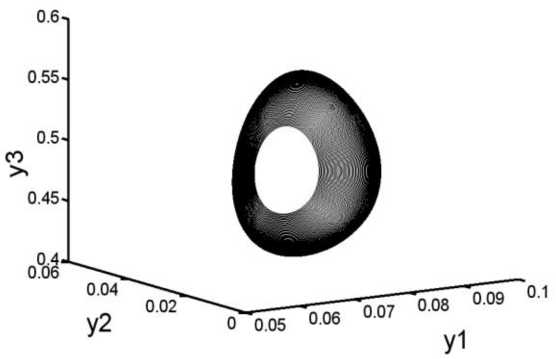
The nontrivial positive equilibrium point of does not exist. For this data, the Fig 1 shows the convergence of trajectories to a non-chaotic periodic attractor showing the long time persistence.

Fig.1. Phase Plot, Time Series and Their Long Time Behavior For Data (10)

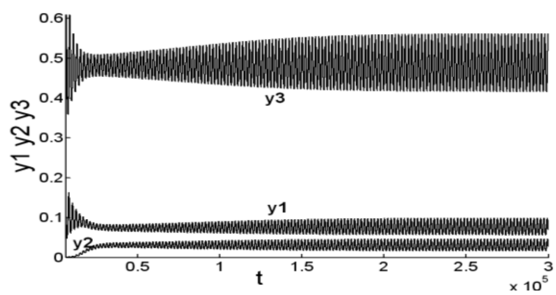
(A) Phase Plot



(b) Long Time Attractor



(C) Time Series



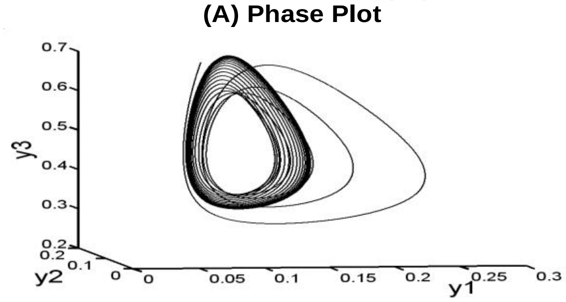
For the following data the two subsystems are also Kolmogorov but only one of them admits stable limit point:

w_1 =3.3, w_2 =1.2, w_3 =1.3, w_4 =1.1, w_5 =2.5, w_6 =1.0, w_7 =1.48, α_1 =3.5, α_2 =2.5 (11)

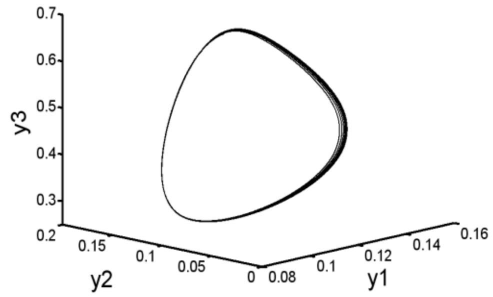
The nontrivial positive equilibrium point of does not exist in this also. Fig.2 shows the existence of a quasi periodic attractor. All the three species coexist.

Fig. 2 Phase Plot, Time Series and Their Long Time Behavior for (11)

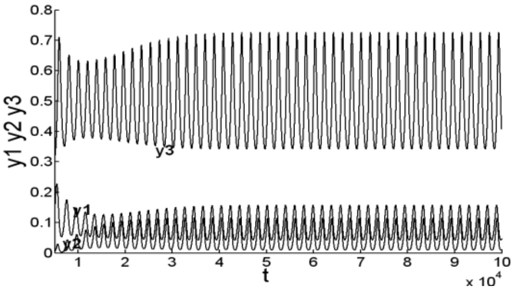
(A) Phase Plot



(b) Long Time Attractor



(C) Time Series



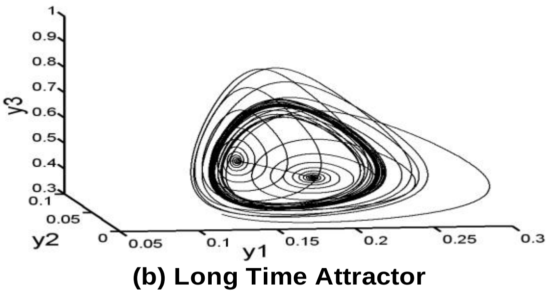
For the following data the two subsystems are Kolmogorov and both subsystems admit limit cycles in their respective planes:

w_1 =5.4, w_2 =3.0, w_3 =3.5, w_4 =1.2, w_5 =3.5, w_6 =1.0, w_7 = 2.4, α_1 =2.7, α_2 =2.0; (12)

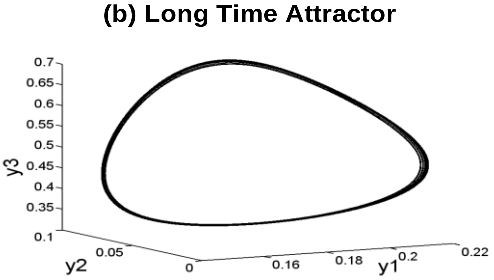
The nontrivial positive equilibrium point of does not exist. The trajectories in fig.3 show the convergence to non-chaotic periodic attractor. The predator switching the preys is clearly visible in the figure.

Fig. 3 Phase Plot, Long Time Behavior and Time Series for (12)

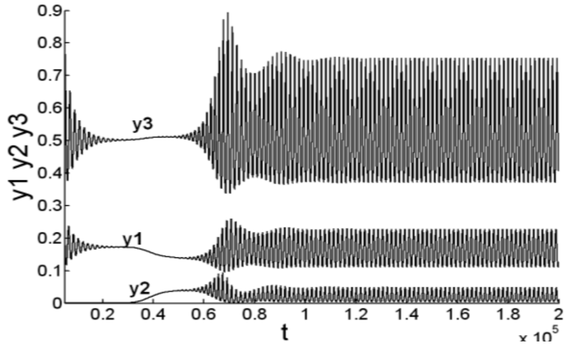
(A) Phase Plot



(b) Long Time Attractor



(C) Time Series



The following data is used in (13):

w_1 =3.9, w_2 =1.4, w_3 =1.5 w_4 =1.1, w_5 =2.4, w_6 =1.0, w_7 = 1.1, α_1 =3.5, α_2 =2.5 (13)

Although, the two Kolmogorov subsystems admit limit cycles in their respective planes and the nontrivial positive equilibrium point of does not exist but the trajectories are quite different here. They also show the quasi-periodic nature of the solution.

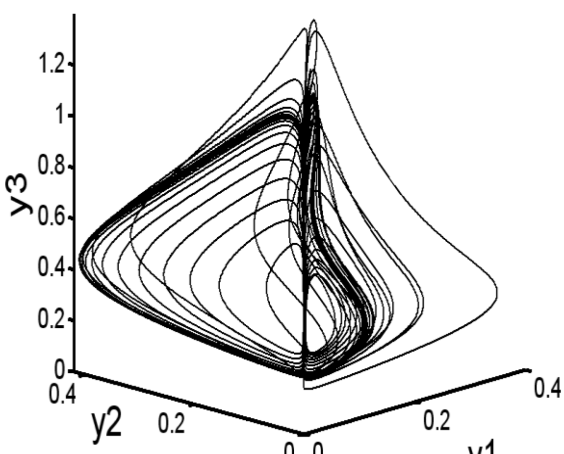
In Fig.4, the trajectories are drawn for the following data:

w_1 = 4.6, w_2 =3.0, w_3 =1.2, w_4 =1.2, w_5 =3.5, w_6 =1.0, w_7 = 2.4, α_1 =2.7, α_2 =1.0; (14)

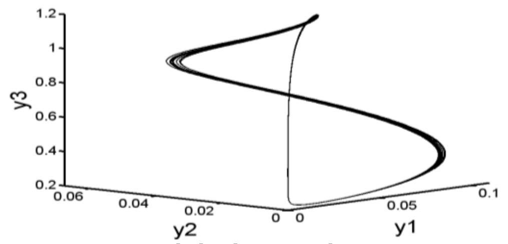
For this data, only one plane admits limit cycle. The mathematical model admits a nonzero positive equilibrium point, which is unstable. In Fig.4 (a), initially the trajectory seems to converge to a quasi-periodic solution but later the loss of stability takes place and solution finally goes to another quasi-periodic solution. Fig 4(b) shows the phase space trajectories after eliminating the initial transients. Time series is shown in Fig 4(c)

Fig.4. Phase plot, long time behavior and time series for (14)

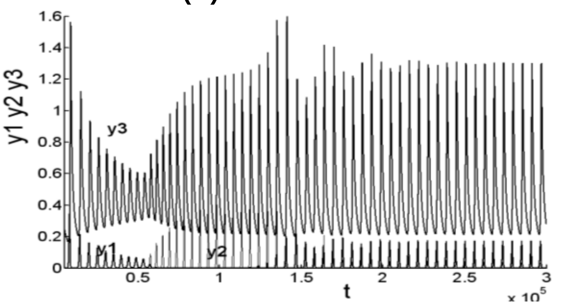
(a) Phase Plot



(b) Long Time Attractor



(C) Time Series



In Fig 5, the trajectories are drawn for the following data:

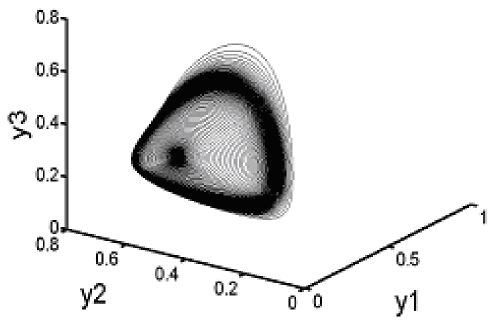
w_1 =3.3, w_2 =1.2, w_3 =1.3, w_4 =1.1, w_5 = 2.5,

w_6 =1.0, w_7 =1.7, α_1 =1.5, α_2 =0.5; (15)

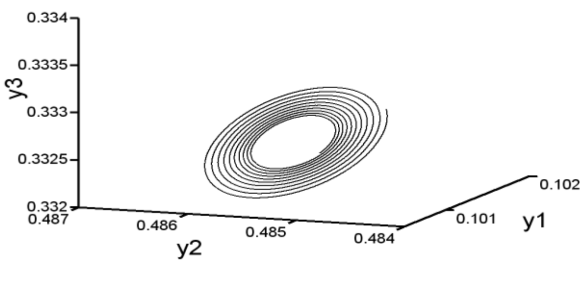
In this case also, only one plane admits limit cycle. The mathematical model admits a nonzero positive equilibrium point, which is stable.

Fig.5. Phase Plot, Long Time Behavior and Time Series for The Data (15)

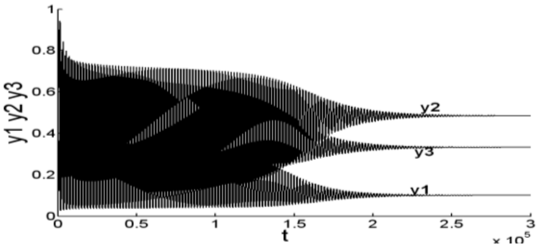
(A) Phase Plot



(B) Long Time Attractor



(C) Time Series



Although the two preys are not directly helping each other, the presence of alternate food to the predator enhances the chances of coexistence of all the three species. It has been observed that the preys can survive at very low densities. At a very low density of first prey, the predator takes food from the second prey. As a result the predator also survives and the second prey decreases while the first prey gets chance to increase its density. Similarly, at low densities of the second prey, the predator survives on the first prey. This may lead to coexistence of all the three species. The predator behaves as a controller in the system. The coexistence in the form of long time quasi-periodic attractor is obtained in variety of cases.

Conclusion

Further, it is observed that quasi-periodic behavior is obtained instead of limit cycle due to relaxation of the constraint considered.Numerical integration of the food-web non-linear system is carried out under the Kolmogorov biologically feasible conditions. Long time behavior of the solution of non-linear system is investigated as non-chaotic quasi-periodic attractor. Due to switching, the persistence of the three species in the form of periodic attractor is possible. However when this assumption is relaxed the quasi periodic behavior is frequently observed. The chaos is not frequently observed and the models reveal quasi periodic nature of the solution. Due to indirect competition between two predator species, one or more species may undergo extinction.

References

1. *Brahampal Singh and Gakkhar, S., The Global Dynamical Behavior of Food Web of Two Logistic prey and Modified Leslie- Gower type Predator, Asean Resonance Journal, P: ISSN No. 0976-8602 RNI No.UPENG/2012/42622 , E: ISSN No. 2349-9443: page no:33-39, Volume- 8, Issue -1, January 2019.*
2. *Brahampal Singh and Gakkhar, S., The Hopf Bifurcation Dynamics of Food Web of Two Logistic prey and Modified Leslie- Gower type Predator, "ShrinkhlaEkShodhparakVaicharikPatrika" (P-ISSN : 2321-290X, E-ISSN : 2349-980X, RNI No. : UPBIL / 2013 / 55327, Scientific Journal Impact Factor : 5.921 , Global Impact Factor 2015: 0.543, International Journal Impact Factor (ISJIF): 6.134). (Volume 06, Issue 09) may 2019.*
3. *Gakkhar, S. and Naji, R. K., Existence of chaos in two-prey, one-predator system, Chaos, Solitons & fractals, Vol. 17, 639-649 (2003).*
4. *Gakkhar, S. and Naji, R. K., On a food web consisting of a specialist and a generalist*

predator, Journal of biological Systems, Vol. 11, 365-376 (2003).

5. *Gakkhar, S. and Naji, R. K., Order and chaos in a food web consisting of a predator and two independent preys, Communications to Nonlinear Science and Numerical Simulations, Vol. 10 ,105-120 (2005).*
6. *Gakkhar, S. and Singh, B., Complex Dynamics in a Food Web Consisting of Two Preys and a Predator, Chaos, Solitons and Fractals Vol. 24, 779-801 (2005).*
7. *Hassard, B.D., Kazarinoff, N.D. and Wan, Y.H. Theory and Applications of Hopf bifurcation. (Cambridge University Press, 1981).*
8. *Hsu, S.B., Hwang, T.W. andKuang, Y. , Rich dynamics of ratio-dependent one-prey two-predators model, Journal of Mathematical Biology , Vol. 43, 377-396 (2001).*
9. *Hutson, V. and Vickers, G.T., A Criterion for Permanent Coexistence of Species, with an Application to a Two-Prey One-Predator System, Mathematical Biosciences, Vol. 63, G.T. 253-269 (1983).*
10. *Jon D.Pelletier, Are large complex ecosystems more unstable? A theoretical reassessment with predator switching, Mathematical Biosciences 163:91-96 (2000).*
11. *Kesh, D., Sarkar, A.K. and Roy, A.B., Succession processes in a food web of a two autotrophy-one herbivore system, BioSystems, Vol. 57, 129-138 (2000).*
12. *Klebanoff, A. and Hastings, A., Chaos in One-Predator, Two-Prey Models: General Results from Bifurcation Theory, Mathematical Biosciences, Vol. 122, 221-233 (1994).*
13. *Koch, A.L., Competitive coexistence of two predators utilizing the same prey under constant environmental conditions, Journal of Theoretical Biology 44: 387-395 (1974).*
14. *Kumar, S., Srivastava, S.K. and Chingakham, P., Hopf bifurcation and stability analysis in a harvested one-predator-two-prey model, Applied Mathematics and Computation, Vol. 129,107-118 (2002).*
15. *May, R. M., Stability and Complexity in Model Ecosystems, (New Jersey Princeton University Press, Princeton, 1974).*
16. *Moiola, J. L. and Chen G, Hopf bifurcation analysis, (World Scientific 1996).*
17. *Owaidy, H.El and Ammar, A.A., Mathematical Analysis of a Food-Web Model, Mathematical Bioscience, Vol. 81,213-227 (1986).*
18. *Peng Feng and Yun Kang; Dynamics of a modified Leslie–Gower model with double Allee effects; Nonlinear Dynamics 80(1-2):1051-1062, April 2015.*
19. *Pimm, S.L., Food webs, (Chapman and Hall, London, England, 1982).*
20. *Polis, G. A. and Winemillr, K. O. (eds) Nonlinear food web models and their response to increased basal productivity, in Food webs: Integration of Patterns and Dynamics, (Chapman and Hall, New York, 1996), 122-133*
21. *Takeuchi, Y., Global dynamical properties of Lotka-Volterra systems, (World Scientific Publishing Co. Pte. Ltd, 1996).*
22. *Yu, P., Simplest normal forms of Hopf and generalized Hopf bifurcations, International Journal of Bifurcation and Chaos Vol. 9, 1917-1939 (1999).*
23. *Zizhen Zhang,Ranjit Kumar Upadhyayand JyotiskaDatta; Bifurcation analysis of a modified Leslie –Gower model Holling Type-IV functional response and nonlinear prey harvesting, Advances in Difference Equations (a open springer journal) 2018 (1), April 2018.*